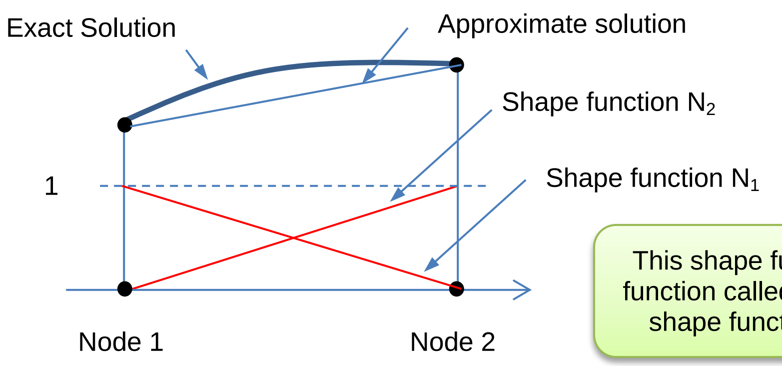
CHAPTER 06: SHAPE FUNCTIONS AND GRAPHICAL EXPLANATION

SHAPE FUNCTIONS

Now let's talk a little bit about the shape function.

The below-mentioned website describe the shape function as:

> *The shape function is the function which interpolates the solution between the discrete values obtained at the mesh nodes. (*http://www.iue.tuwien.ac.at/phd/orio/node48.html)



This shape function is one of common shape function called piece-wise linear. However, the shape function does not need to be linear.

The above image shows a shape function corresponding to node 1. Recall the previous chapter (for example, W consists of shape functions as shown in the following equation) there will be N number of shape functions.

$$W = d_1N_1 + d_2N_2 + \ldots + d_nN_n$$

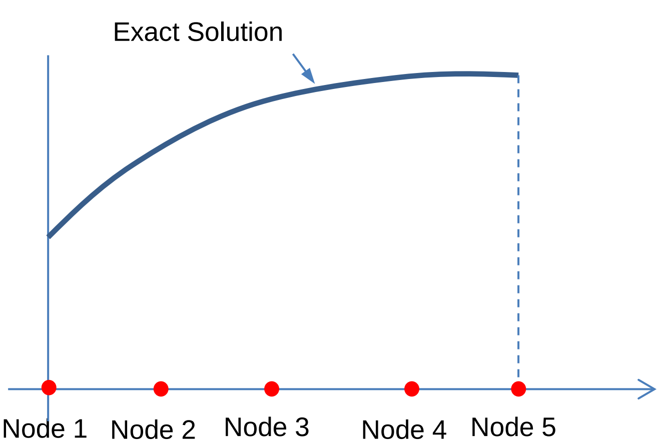
Because of linear property, it is easy if (in fact it needs to be that) a shape function has a value of 1 at its corresponding node, and zero elsewhere.

Shape Function Property

$$N_A(x) = 1 \text{ for } x = A$$
$$N_A(x) = 0 \text{ for } x \neq A$$

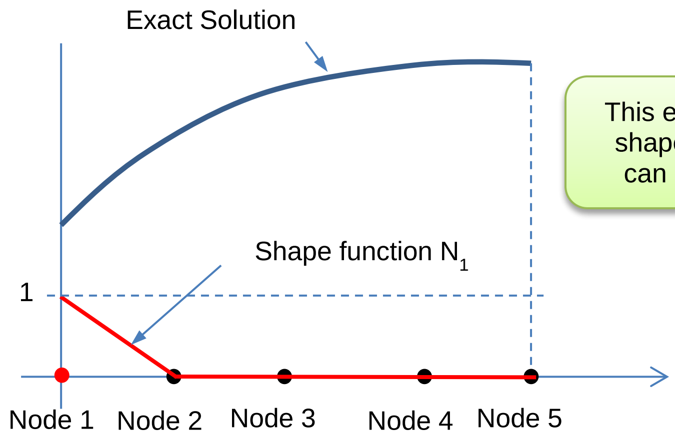
GRAPHICAL IDEA

Suppose we want to approximate a solution with 5 nodes (4 elements) as shown below.



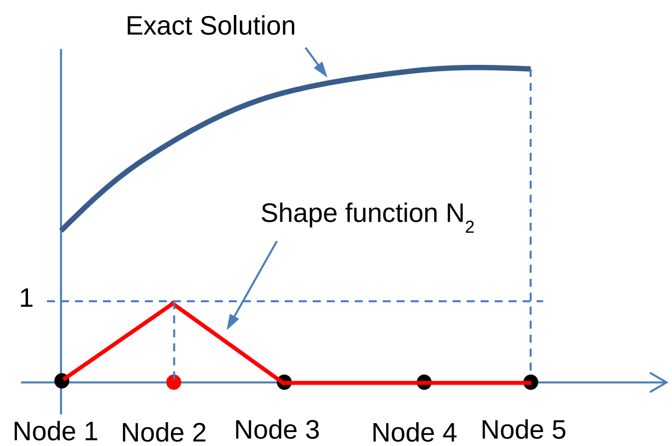
We'll use piece-wise linear shape function as shown in the above example.

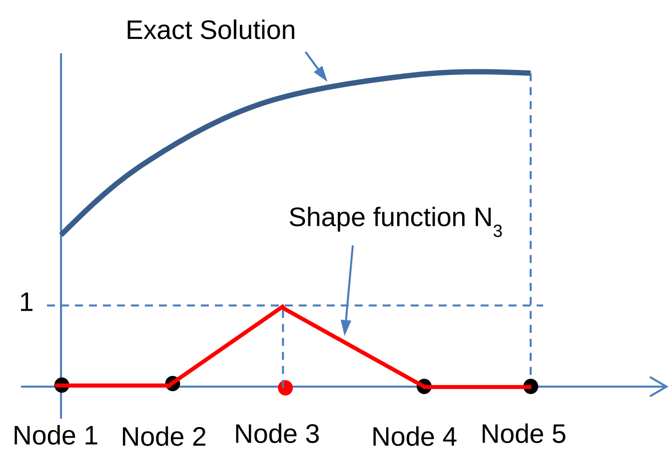
Remember, the shape function has a value of 1 at its node, and zero elsewhere. Therefore, the first shape function N_1 will be like this.

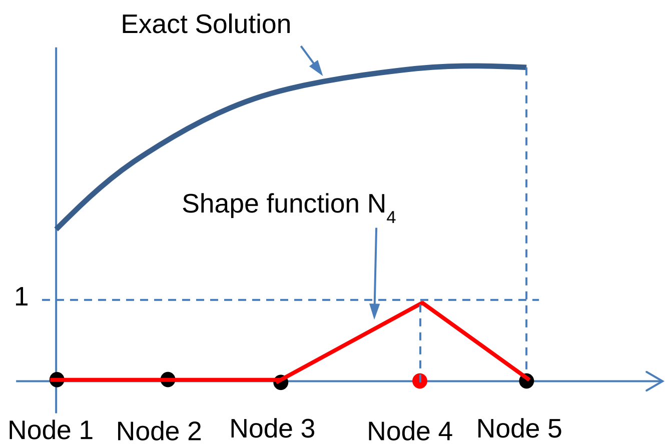


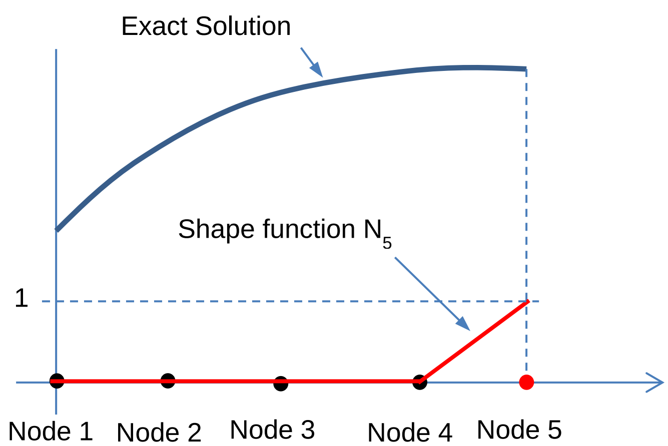
This example uses "piece-wise linear" shape function as an example. FEM can use other shape functions too.

Similarly the other shape functions are:

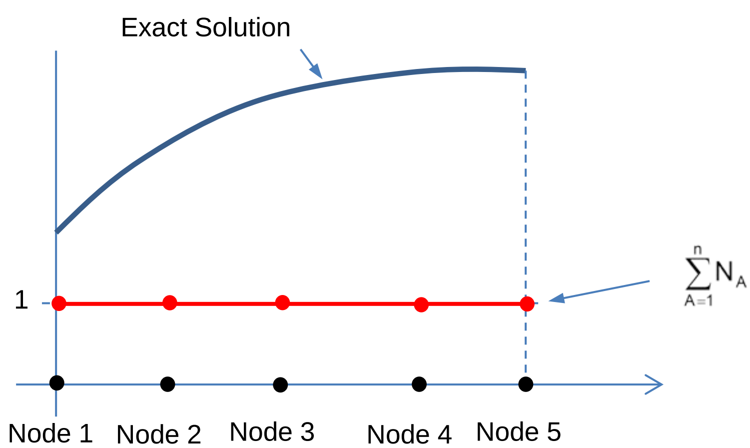




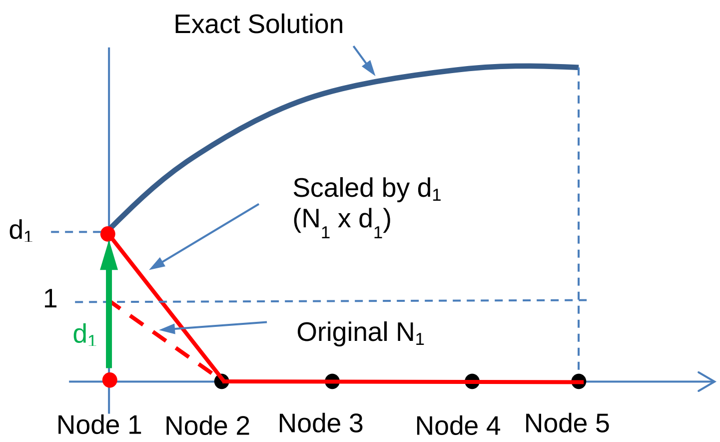




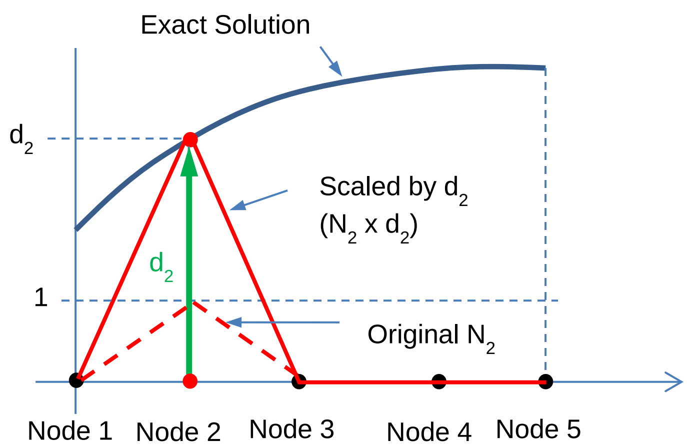
If you add all shape functions, it will become "one" at all nodes like this. This is not an approximate solution yet, of course.

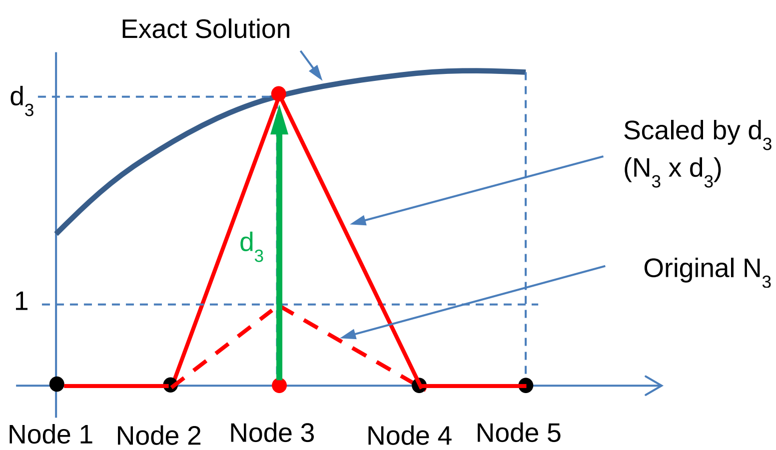


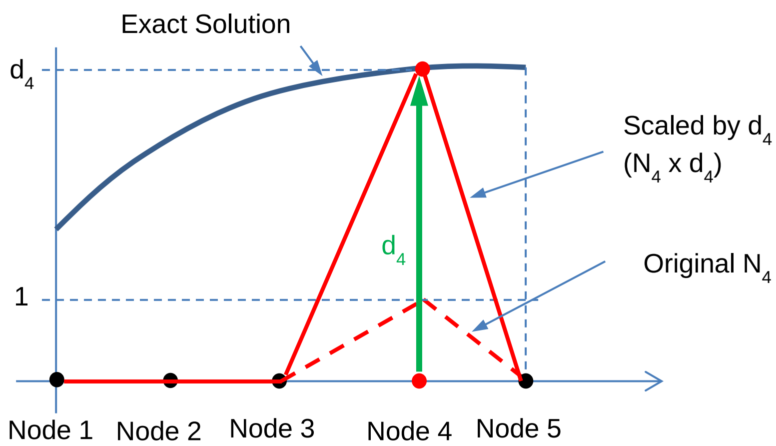
Every node must be "scaled" by "d_n" to make it an approximate solution. That is, for example using node 1,

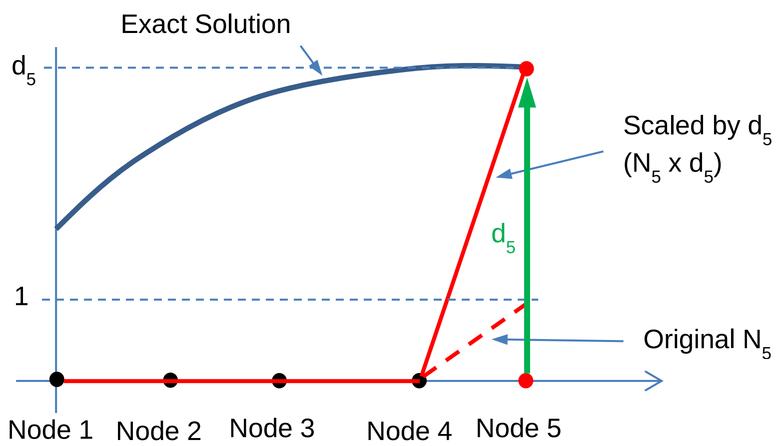


Similarly, the shape functions multiplied by "d" (displacement vectors) for 2nd node and on are:

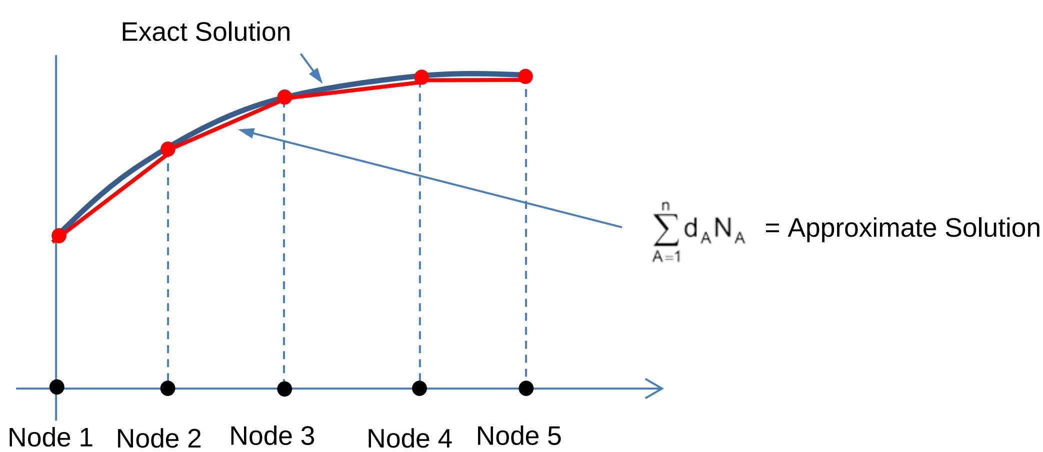








If you add all shape functions multiplied by d, it will become an approximate solution.



In the above example, shape functions are known. Only the unknowns are "d_n" (displacement vectors). If you can find displacement vectors, you can find an approximate solution. Therefore, FEM problems are, in other words, how to solve for the displacement vectors.