CHAPTER 04: GALERKIN METHOD

GENERAL EXPRESSION OF GALERKIN METHOD

Let's write down Eq.03-4 again.

$$\int_\Omega W_A(u-\varphi)d\Omega = \sum_{A=1}^{n} a_A \int_\Omega W_B N_A d\Omega \qquad \text{Eq.04-1}$$

Galerkin Method uses a derivative of approximate function $\hat{u}$ with respect to unknown coefficient a, as the **weighing function** W. This means,

$$W = \frac{\partial}{\partial a}\hat{u}$$

Because $\hat{u} = \varphi + \sum_{A=1}^{n} a_A N_A$ (from Eq.03-1), the W_A (W that corresponds to a specific node "A") can be written as follows:

$$W_A = \frac{\partial}{\partial a_A} a_A N_A = N_A$$

Therefore,

$$W_A = N_A \qquad \text{Eq.04-2}$$

In shorts, **Galerkin Method** uses N (**shape function**) for W (**weighting function**).

Inserting Eq.04-2 into Eq.04-1, then,

$$\int_\Omega N_A(u-\varphi)d\Omega = \sum_{A=1}^{n} a_A \int_\Omega N_A N_B d\Omega \qquad \text{Eq.04-3}$$

The term $\sum_{A=1}^{n} a_A$ is a vector form and can be expressed as $\begin{Bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{Bmatrix}$ and called **displacement vector**, or $\bar{d}$.

The term $\int_\Omega N_A N_B d\Omega$ forms n x n matrix form and expressed as K_{AB}. This is called **stiffness matrix**.

Finally, the LHS term is called **force vector** and expressed as $\bar{f}$.

Therefore, Eq.04-3 can be shown as:

$\bar{f} = K_{AB}\bar{d}$, or simply f=Kd.

$$\underbrace{\int_\Omega N_A(u-\varphi)d\Omega}_{\bar{f}} = \underbrace{\sum_{A=1}^{n} a_A}_{\bar{d}} \underbrace{\int_\Omega N_A N_B d\Omega}_{K_{AB}}$$

LEAST SQUARE METHOD (OFF THE TOPIC)

Galerkin Method is one of methods among **Method of Weighted Residual**. What if we choose something else for the weighing function W?

Suppose we choose the derivative of residual R with respect to unknown coefficient a as weighing function.

That is,

$$W_A = \frac{\partial}{\partial a_A} R$$

The LH side of Eq.03-3 becomes

$$\int_\Omega R W d\Omega = \int_\Omega R \frac{\partial}{\partial a_A} R d\Omega$$

By the way, if we want to achieve a minimum of the integral of squared R, so-called **Least Square Method**

$$\int_\Omega R^2 d\Omega = 0$$

Comparing this equation with Eq.03-3, we can say that **Least Square Method** is one of the **Method of Weighted Residual** in which the derivative of residual R with respect to unknown coefficient is chosen as weighing function.

Least Square Method is ***not*** FEM to solve structural engineering problem.