CHAPTER 03: METHOD OF WEIGHTED RESIDUAL

MATHEMATICAL THEORY

In the previous chapter we talked about approximate function. One of the mathematical techniques to find the approximate function is called **Method of Weighted Residual** which is a core theory behind the FEM.

Suppose you have a function u and $\hat{u}$ is an approximate function to u.

Then, we can express $\hat{u}$ as follows:

$$\hat{u} = \varphi + \sum_{A=1}^{n} a_A N_A \quad \text{.......Eq.03-1}$$

where, φ = function satisfies all boundary conditions

a_A = unknown coefficient

N_A = known linear function

> N_A is also known as **shape function** or **base function**

Now, let define "residual" R as follows.

$$R = u - \hat{u} \quad \text{.......Eq.03-2}$$

In words, R is the residue of u after subtracted by $\hat{u}$, which is "residual".

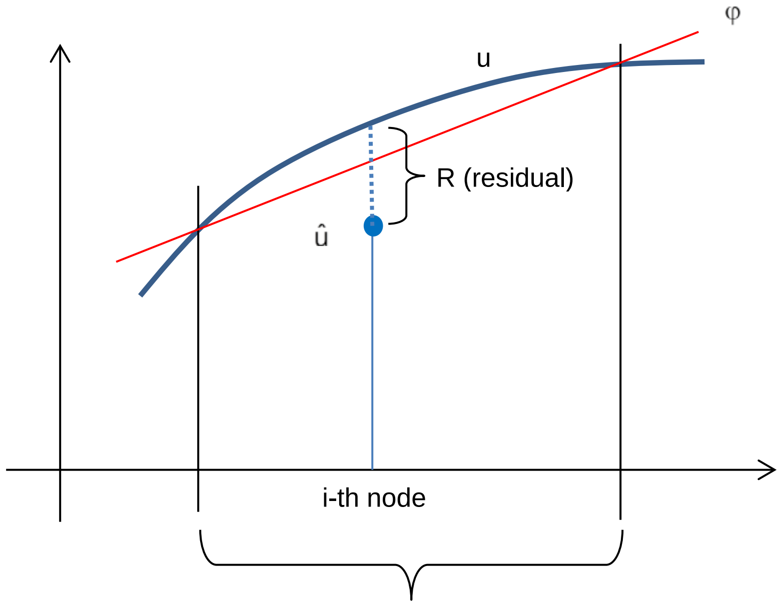
Method of Weighted Residual is to make integral of R multiplied by **Weighing Function** W becomes zero. Therefore,

$$\int_{\Omega} W R \, d\Omega = 0 \quad \text{.......Eq.03-3}$$

> Ω defines a domain of the integral. Governing PDE is a mathematical expression while we are working on finite physical problem. We only care about the approximation of PDE function only on the defined domain, which is Ω.

The reason of multiplying W is to take average sense over the domain.

The figure below shows a graphical idea.



Inserting Eq.03-2 into Eq.03-3, then,

$$\int_\Omega W(u - \hat{u})d\Omega = 0$$

Inserting Eq.03-1, then,

$$\Rightarrow \int_\Omega W_A\left(u - \varphi - \sum_{A=1}^{n} a_A N_A\right)d\Omega = 0$$

Reorganizing it, then,

$$\Rightarrow \int_\Omega W_A(u - \varphi)d\Omega = \sum_{A=1}^{n} a_A \int_\Omega W_B N_A d\Omega \quad \text{.........Eq.03-4}$$

Note that the subscript of W on RHS changed to B (W_B) in order to distinguish from the original subscript of N (N_A). This is because for each A of N, there is B of W.

Then, what is W (**weighing function)**? Let's move to the next chapter to explain this.