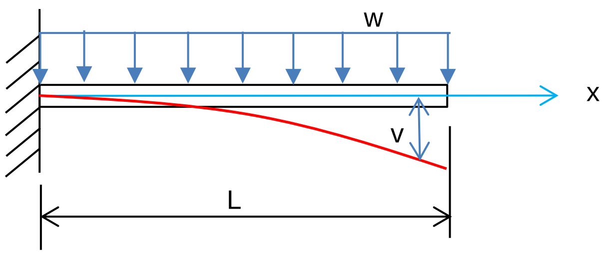
CHAPTER 02: APPROXIMATION OF FUNCTION

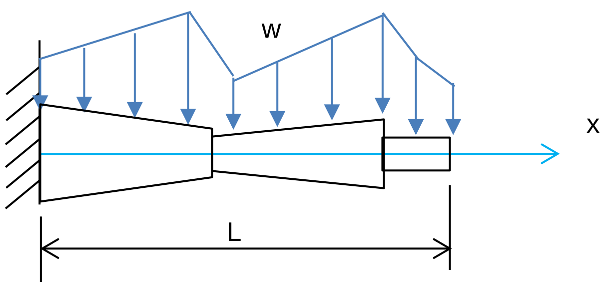
WHAT IS FEM?

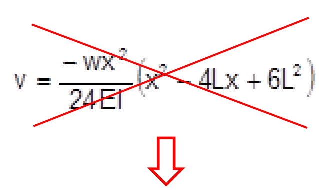
FEM stands for Finite Element Method. Engineering problems are in the form of **Partial Eifferential Equations (PDE)** with boundary condition. However, in almost all times in a real world you cannot find exact solutions, or even though it is possible, it is very difficult. FEM is a technic to find approximate solution to partial differential equations by breaking it into pieces (i.e., finite elements). Here is an example. From Euler-Bernoulli's beam theory, we have the following *exact* solution to the uniform loaded, cantilever problem as shown in the figure below.

Deflection $v = \frac{-wx^2}{24EI}\left(x^2 - 4Lx + 6L^2\right)$ Eq.02-1



The above equation assumes that E, I, and w values are constant throughout the beam. Unfortunately, in the real world it is not that simple. What if we have a similar problem but w, E, and I all vary?

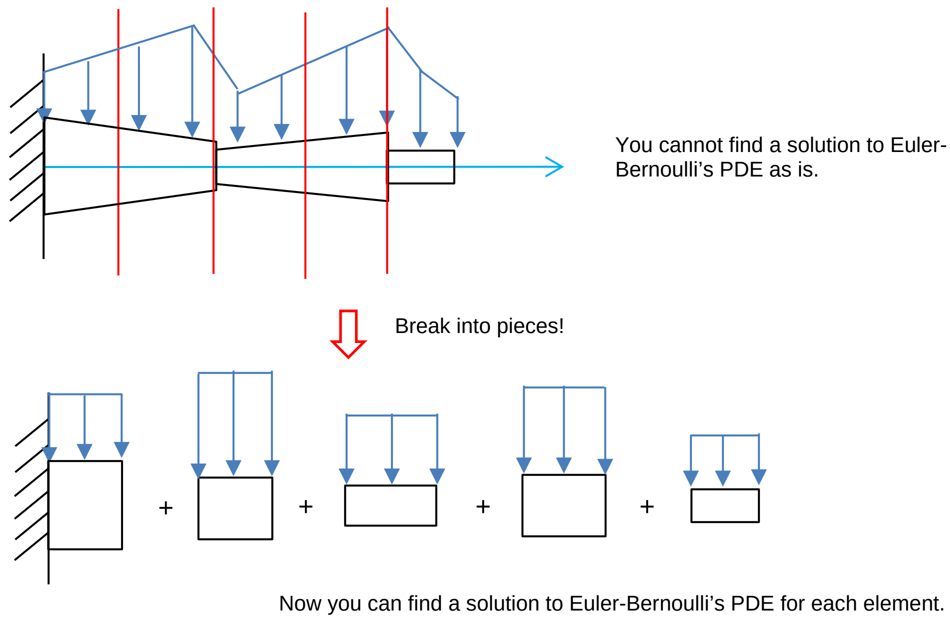




No exact solution can be found

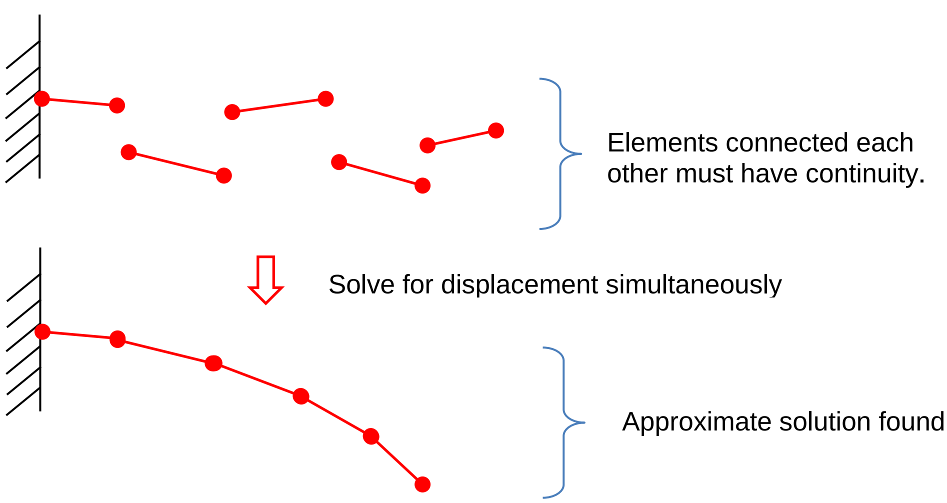
Obviously you cannot use equation Eq.02-1 anymore.

However, if you break the beam into small pieces where changes of w, E, and I are reasonably small within each broken piece, then you may be able to find exact solution on each small piece. (The below figure is still coarse and not small enough.) Since you will be breaking it into whatever a number of elements but finite element numbers, this technique is called Finite Element Method, or FEM.



Because you broke the beam into many pieces, you need to connect them again. To do this, the displacements at the both ends of each piece must agree with these of the elements next to it. Otherwise, there will be a "discontinuity" between the elements and solution will not be correct. These displacements are unknowns and you will be solving them simultaneously over all of the elements so that you can eventually bring the separated elements into one original beam.

Once you can successfully reassemble the elements, the beam is now called "approximate solution".

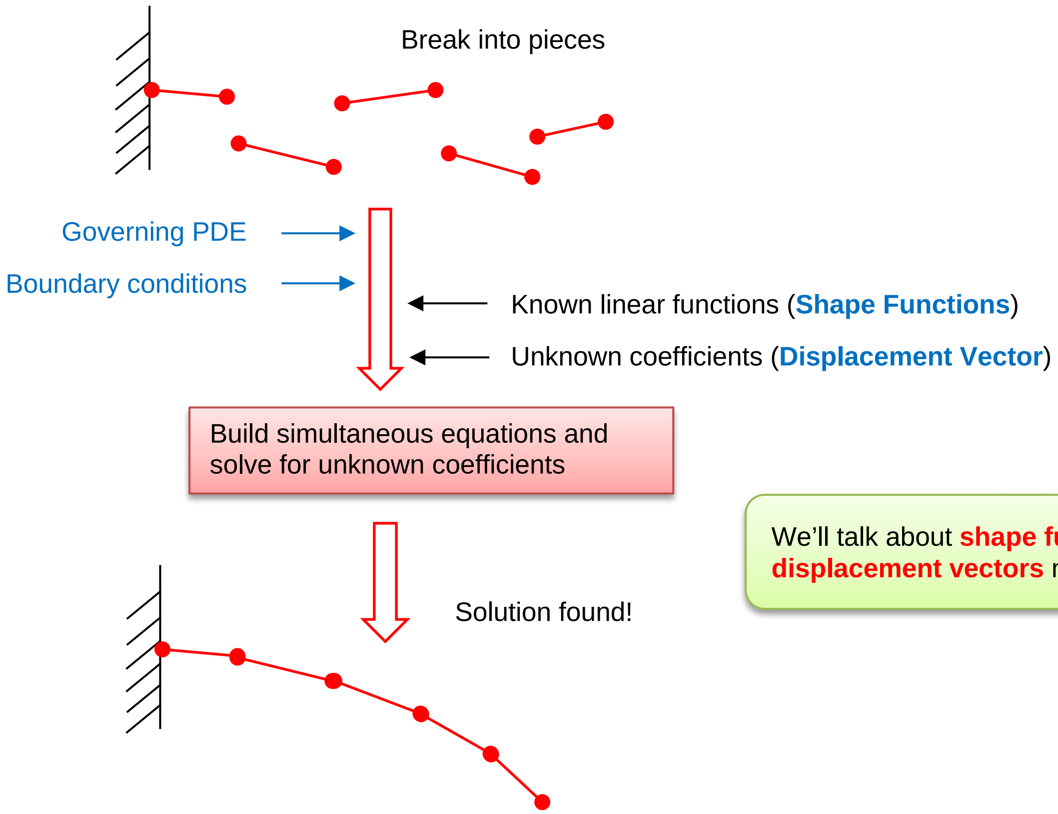


The above example is to give you a sense of idea about FEM. Please do not worry if you don't understand it. We'll actually solve this FEM problem by hand in the later chapter.

APPROXIMATED FUNCTION

In the previous paragraph I have mentioned about "approximate solution". So, what is it? How to approximate a solution?

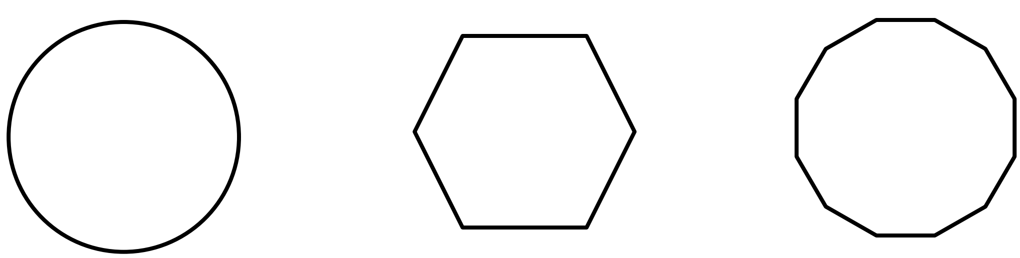
In FEM, the approximation is done by a sum of linear functions multiplied by coefficients. Linear functions are known functions with your choice, called shape functions. The coefficients are unknowns and called displacement vectors. In the above example, displacements are unknown coefficients. Again, these will be explained in more detail later. For now, just get an image.



We'll talk about **shape functions** and **displacement vectors** more later.

The mathematic technique to find approximate solution, which is a core of FEM theory, is called **Method of Weighted Residual**. We'll talk about it in the next chapter but for now let's talk a little bit more about approximate solution.

In general engineering PDE problems are considered as mathematical functions. Therefore, I use "approximate function" for the same meaning as "approximate solution". Below examples show comparisons of exact function and approximate function. A circle is your exact function and there shows two approximate functions; one with 6 linear elements and the other with 12 linear elements. Of course, the more you have elements, the more accurate the approximate function is.



Exact function (Perfect circle) | Approximation by 6 linear elements | Approximation by 12 linear elements